Equilibrium and dynamical properties of few particle systems in single and coupled e-h-quantum dots

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1 Introduction

- Mesoscopic electron systems in quantum dots are known to reveal interesting correlation effects, e.g. Wigner crystallization [2]
- **Extensions in present work**: 1. Single-electron control of melting/conductivity [1] confirmed by dynamic simulation. 2. Generalization to mesoscopic electron-hole *bilayers* (vertically coupled quantum dots): crystallization of excitons observed, energy spectrum is found.

2 Single-electron control of circular current in quantum dots

- Investigation of the response of mesoscopic 2D electron clusters to external angular excitation.
- Control of the angular current by addition/removal of a single electron ⇒ application to single-electron devices [1].
- Idea: use the striking difference of orientational melting temperatures of cluster with $N = 19, 20$, see Fig. 1, to switch between liquid-like ("conducting") and crystal-like ("insulating") behaviour [2].

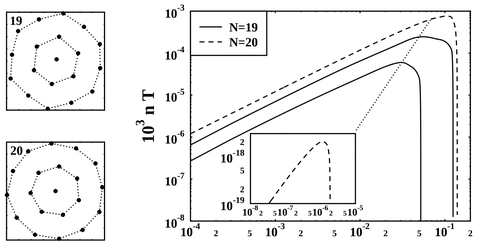
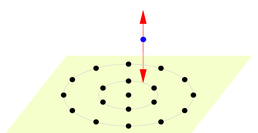


Fig. 1 *Left*: Ground state configuration of $N = 19$ and $N = 20$ clusters. *Right*: Phase diagram of the "crystal" phases for $N = 19, 20$. The outer (inner) lines are the radial (angular) melting phase boundaries. The classical melting takes place at specific values of the coupling parameters: $\Gamma_{20}^{RM} = 83, \Gamma_{19}^{RM} = 154, \Gamma_{19}^{OM} = 330$ and $\Gamma_{20}^{OM} = 3.4 \cdot 10^{11}$. $\Gamma = \left(\frac{e^2/\epsilon(a)}{k_B T}\right)$.

2.1 Model and simulation idea



Hamiltonian of a single quantum dot:

$$\hat{H} = -\sum_{i=1}^{N} \frac{\hbar^2 \nabla_i^2}{2m_i} + \sum_{i=1}^{N} \frac{m_i \omega_0^2 r_i^2}{2} + \sum_{i<j}^{N} \frac{e^2}{\epsilon |\mathbf{r}_i - \mathbf{r}_j|}$$

After reaching the ground state configuration (see above) the system is disturbed by: (1) *angular external force*, (2) friction. We calculate the response by solving:

$$m_e \ddot{\vec{v}}_i(t) = \sum_{i,j} \vec{F}_{ij} + \vec{F}_i^{ext} - \gamma \vec{v}_i(t), \quad i = 1, \dots N$$

- External force, $\vec{F}_i^{ext} = const$, acts only on the outer shell.
- To prevent rotation of the cluster as whole we pinned *one particle of the inner shell*, it allows to move only radially.
- One particle is *slowly* added/removed to the inner shell.

Result: **I)** Angular current (intershell rotation) flows in $N = 20$ cluster and stops after removal of 20-th particle.

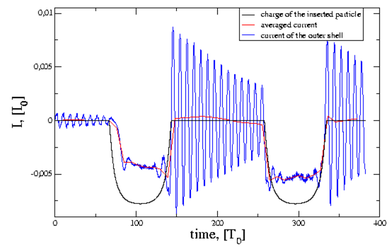
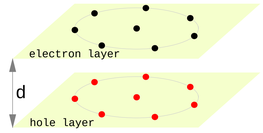


Fig. 2: Dependence of the outer shell current on time. Averaged current is also shown. Black line is the charge of the inserted particle. Time is in units of $T_0 = 2\pi/\omega_0$. Units of current is $I_0 = e\frac{\Gamma_0}{T_0}$, with unit of length $r_0 = \left(\frac{2e^2}{e m \omega_0^2}\right)^{1/3}$.

II) We demonstrate principial possibility of creation the device with open and close states. The inserted particle should be inserted about 30 T_0, friction coefficient should be choosen to ensure steady constant current in open state.

3 Mesoscopic electron-hole bilayer systems



Hamiltonian of two vertically coupled electron-hole QD's:

$$\hat{H} = H_e + H_h - \sum_{i=1}^{N_e} \sum_{j=1}^{N_h} \frac{e_i e_j}{\epsilon \sqrt{|\mathbf{r_i} - \mathbf{r_j}|^2 + d^2}},$$

$$\hat{H}_{e(h)} = \sum_{i=1}^{N_{e(h)}} \left[-\frac{\hbar^2}{2m_i} \nabla^2 + V_{e(h)}(\mathbf{r}_i) + \sum_{i<j}^{N_{e(h)}} \frac{e_i e_j}{\epsilon |\mathbf{r_i} - \mathbf{r_j}|} \right]$$

We consider symmetrical bilayers: $m_e = m_h = m$.

Confinement potentials are equal: $V_e = V_h = \frac{1}{2} m \omega^2 r^2$.

The state of the system is governed by two competing effects:

- *Intra-layer* Coulomb repulsion of particles → "crystallization" in each layer
- *Inter-layer* attraction between electrons and holes → formation of inter-well excitons (or dipoles in the classical limit)

3.1 Classical mesoscopic bilayers

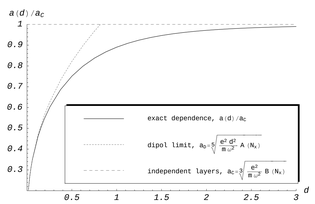


Fig. 4 Intra-layer particle distance, $a(d)$, in the ground state as a function of layer seperation d (the unit of length is a_c). We have two limits: (I) $d \ll 1$, $a(d)$ is determined only by dipole-dipole interaction (mean interparticle distance a_D), (II) $d \gg 1$, intra-layer Coulomb interaction dominates (mean distance a_C). Note the different power laws of $a(d)$ in the two limits.

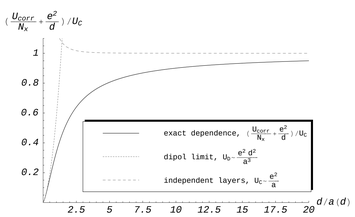


Fig. 5 Correlation energy (binding energy of excitons is excluded) for different distances between layers, d. Figure compares the exact correlation energy with the two limits: system of dipoles, U_D, and independent layers, U_C.

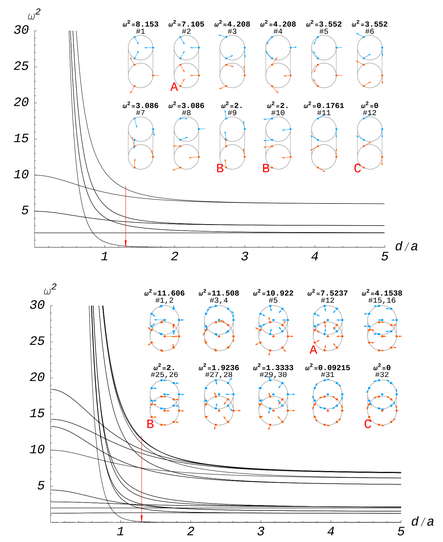


Fig. 6, 7 Excitation spectrum of normal modes for three and eight excitons as a function of distance d. The eigenfrequencies, ω, are in units of the trap frequency divided by $\sqrt{2}$. There are always $2N$ normal modes. Insets display the eigenvectors and eigenvalues for $d/a = 1.3$ (in descending order) computed from the Hesse-Matrix: $H_{ij} = \partial^2 H / \partial r_i \partial r_j$, $\vec{r} = \{x_1, y_1, \dots x_N, y_N\}$ is a vector of all particle coordinates.

For all systems one always finds three trivial, universal modes:

A) *rotation of the whole system*, $\omega^2 = 0$,

B) *vibration of the center of mass*, $\omega^2 = 2$ (two-fold degenerate),

C) *breathing mode*, exact result for general interaction of the form $U \sim \frac{e^2}{a^{1+p}}$:

$$\omega_{br}^2 = 2m(3+p) = \begin{cases} 6, & p = 0 \quad \text{Coulomb} \\ 10, & p = 2 \quad \text{dipole} \\ 14, & p = 4 \quad \text{quadrupole} \end{cases}$$

For excitons ω_{br}^2=20 (dipole interaction, $m = m_e + m_h$).

Result: Analysis of mode spectrum reveals:

I) $d > 1$: anti-phase rotation of shells in the electron and hole layers has the lowest energy [mode 11 (Fig. 6) and 31 (Fig. 7)] ⇒ with increasing temperature/density the two layers become decoupled and excitons are destroyed.

II) $d < 1$: lowest excitation energies are related to pairwise coupled motion of electrons and holes ⇒ crystal melting proceeds via transition from exciton crystal to exciton liquid (excitons are preserved).

3.2 Quantum mesoscopic bilayers

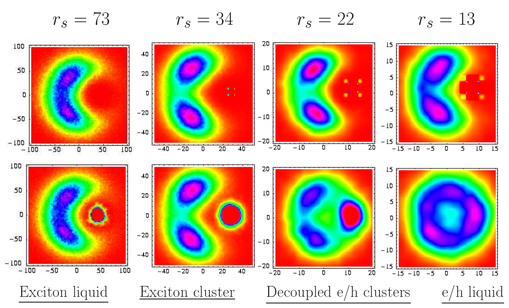


Fig. 5 Pair correlation function for 3 electrons and 3 holes (position of one electron is fixed), inter-layer distance $d = 10 a_B$, temperature $T \leq 1/7 \times 10^{-3}$ Ha.

Results for temperature and density boundaries for the existence of e-h crystal: $T \lesssim 1/7 \times 10^{-3}$ Ha, $r_s \gtrsim 20$.

Examples:

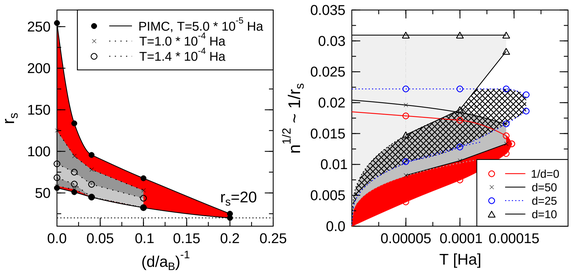
a) **GaAs** -based structures: $T \lesssim 40\,mK$ and $\rho \lesssim 8 \times 10^8\,cm^{-2}$

b)**CdTe** -based structures: $T \lesssim 100\,mK$ and $\rho \lesssim 9 \times 10^9\,cm^{-2}$

c) **ZnSe** -based structures: $T \lesssim 400\,mK$ and $\rho \lesssim 3 \times 10^9\,cm^{-2}$

3.3 Phase diagramm of e-h bilayers

1. *Exciton crystal* can exist only for inter-layer distances $d \gtrsim 5a_B$ and critical density parameter $r_s \gtrsim 20$.
2. Interlayer attraction leads to stabilization of crystal phase: e.g., for exciton cluster, $N_e = N_h = 8$, coupling parameter r_s changes from $r_s \approx 56$ (for $d \to \infty$) to $r_s \approx 20$ (at $d = 5a_B$).
3. Highest melting temperature is observed for intermediate $d \sim 25a_B$.



4 Summary and Outlook

⇒ Single-electron control of melting/conductivity is demonstrated.

⇒ Evidence for crystallization of excitons is found (not possible in single layers or for $d \lesssim 5a_B$).

⇒ Two crystal phases exist: exciton and decoupled e/h crystals.

⇒ Phase diagram (in T, n, d space) of mesoscopic e-h systems is presented.

⇒ (Classical) Excitation spectrum has been found and crystal melting scenario clarified.

⇒ Bose condensation of mesoscopic excitons expected at higher densities.

References

[1] M. Bonitz, V. Golubnychij, A.V. Filinov, Yu.E. Lozovik, Microelectronic Engineering **63**, 141 (2002)

[2] A. Filinov, M. Bonitz and Yu.E. Lozovik, Phys. Rev. Lett. **86**, 3851 (2002)

[3] An analysis of classical macroscopic e-h bilayers is performed in: P. Hartmann, Z. Donko, and G. Kalman, submitted to Phys. Rev. Lett.

[4] Quantum macroscopic e-h bilayers has been invesitaged in: S. De Palo, F. Rapisarda, and G. Senatore, Phys. Rev. Lett. **88**, 206401 (2002)

[5] G. Goldoni and F.M. Peeters, Phys. Rev. B **53**, 4591 (1996); I.V. Schweigert, V.A. Schweigert, and F.M. Peeters, Phys. Rev. Lett. **82**, 5293 (1999)