Wake formation and wake field effects in complex plasmas

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Received XXXX, revised XXXX, accepted XXXX
Published online XXXX

Key words wake field, grain charging, simulation, experiment, complex plasma

The formation of wake fields downstream of an object in flowing plasmas has strong implications on structure, stability and dynamics of complex plasmas. This paper aims at putting recent experimental investigations and different theoretical approaches on charging of dust grains, stability of dust grain arrangements and the dynamical properties of complex plasmas into perspective. Further, the combination of wake fields and grain confinement is discussed, which is the generic situation in complex plasmas. It is shown that in spherical traps the resulting competition of trap geometry and vertical alignment leads to a sophisticated mixture of nested shell and string order.

1 Introduction

Symmetry plays a fundamental role in physics. The interaction of dust grains in a complex plasma is generally adequately described by an isotropic and spherically symmetric Yukawa potential. This symmetry is lost as soon as plasma and dust grains move relative to each other. Thus, studies of objects in streaming plasmas are interesting from a very fundamental point of view. The broken symmetry is known to have a large impact on structure, stability and dynamical properties of this strongly coupled model system [1–3]. Non-equilibrium and streaming effects are omnipresent in nature and play an important role in many laboratory situations. Therefore, a profound understanding of objects in streaming plasmas is essential for a variety of problems in plasma physics, e.g. processes in the plasma sheath, probe theory, and measurements near spacecrafts. However, it is the field of complex plasmas that provides unique opportunities to investigate these basic processes in detail [4].

In laboratory plasmas strong ion flows are typically found in the plasma sheath. Due to a balance of gravitational and electric field forces dust grains are typically confined in this region. Thus, the plasma sheath region is ideally suited to study the interaction of dust grains in supersonic ion flows. Already with the first attempts to create three dimensional dust crystals it turned out, that grains tend to align in the direction of the ion flow. The reason for this alignment is that the upstream (highly negatively charged) grain acts as an electrostatic lens, which focuses the flowing ions downstream in a distance of about one electron Debye length λ_{De}. The resulting positive space charge region attracts downstream grains, which thus favor an aligned position. The existence and characteristics of the attractive wake field has been studied by a number of analytical and numerical studies [5–21] and experiments [22–29]. Based on systems consisting of two grains the wake field topology [27], ion focus strength [23, 25, 26] and influence of the confinement [24, 27, 30] have been studied.

While these investigations provide a basic understanding of wake fields in complex plasmas, fundamental questions remained unanswered. Recent progress in experiment [28–30] and simulation [31–33] now allows to

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address these open issues. First, advanced numerical studies of wake fields [21, 31, 33] allow for an one-to-one comparison of different simulation approaches. Second, the formation of ion wakes has been intensively studied for supersonic flows. In contrast, much less attention was paid to the subsonic regime, although the subsonic regime is of high interest for experiments where grains are confined in the pre-sheath region or the plasma bulk [28, 30, 34]. Recently, numerical approaches [31–33, 35] started to focus on wake field formation in this regime. Third, it has been assumed that the charging process for dust grains is not affected by the wake field. Experiments by Kong *et al.* [36] , however, have indicated that the charging process of the dust grains should be affected by the ion flow and the arrangement of the grains. Recent experiments [28, 29] in combination with simulations [33] clearly show that in extended dust clouds identical dust charges are only a rough estimate.

This paper aims at putting the experimental and numerical progress on charging of dust grains, stability of dust grain arrangements and their dynamical properties into perspective and discusses the implications for research on complex plasmas.

2 Wake Fields in Theory, Simulation, and Experiment

A rigorous method to compute the wake field should be based on first principle calculations which solve the Vlasov equations for all plasma constituents and mimic collisions for instance with a Monte-Carlo approach. Powerful tools for this purpose are Molecular Dynamics (MD) simulations and Particle-In-Cell (PIC) methods. Multi-scale MD and PIC simulations are, however, computationally very demanding and limited to the static description of very few grains ($N < 10$). On the other hand, they include nonlinear effects, possible deviations from a shifted Maxwellian ion velocity distribution and allow for treating the charging process of the grains self-consistently. Compared to MD, the PIC method can handle significantly more plasma particles and thus reduces the numerical noise. Recent PIC codes, e.g. SCEPTIC [15], COPTIC [21], a hybrid $\mathrm{P^3M}$-code [37] and DiP3D [16] use three-dimensional grids to simulate the plasma dynamics around dust grains with realistic grain properties (i.e. dimensions, shape and conductance). SCEPTIC and COPTIC simulate ions with the PIC kinetic scheme assuming Boltzmann distributed electrons. The hybrid $\mathrm{P^3M}$-code and DiP3D are full PIC codes, where also the electrons are treated on first principles. Usually, 10^7 plasma particles are simulated around the grain. In addition, the hybrid $\mathrm{P^3M}$-code, DiP3D, and COPTIC allow to compute the charging of multiple grains. To correctly resolve the interaction of dust grains, which have an intergrain distance of the order of the electron Debye length, these codes (COPTIC, DiP3D, hybrid $\mathrm{P^3M}$) use the Particle-Particle-Particle Mesh scheme [38]. Since the relevant time scale for PIC simulations is given by the ion plasma frequency, which is several orders higher than that of the dust grains, the dust grains are generally treated as stationary objects.

Therefore, to access structure and dynamics of dust clouds it is necessary to capture the dust-plasma interaction in a time-averaged (for electrons and ions) model. A common simplification for a complex plasma, consisting of electrons, ions, neutral atoms and dust grains, is the so called one component plasma (OCP) model, where the dust grains are treated individually and all plasma properties are represented by the longitudinal dielectric function $\epsilon(\mathbf{k}, \omega)$. In linear approximation this results in an interaction potential

$$\Phi(\mathbf{r}) = \int d^3k \frac{Q_d}{2\pi^2 k^2\, \epsilon(\mathbf{k}, \mathbf{k}\cdot\mathbf{u_i})}\, e^{i\mathbf{k}\cdot\mathbf{r}}, \tag{1}$$

where Q_d is the dust charge and $\mathbf{r}$ denotes the distance from the grain. For isotropic and stationary plasma conditions, this gives the well known Yukawa potential. However, for a streaming plasma the response of the plasma in direction of the flow is more complicated. Linear Response (LR) theory allows for a high precision numerical computation of the dynamically screened Coulomb potential from the dielectric function. A detailed description of the numerical treatment is given in [31] and references therein. LR allows to study the generic features of wake fields in great detail with low numerical noise over broad parameter ranges, e.g. for collisionless and collisional plasmas [31]. Furthermore, LR provides in combination with Molecular Dynamics (MD) simulations the unique opportunity to study the impact of streaming plasmas on the dynamical, correlated interplay of many dust grains with unrivalled precision (within the linear approximation) [32, 39, 40]. However, this approach assumes point-like (non-absorbing) grains and does not account for self-consistent grain charging processes yet.

Experimental approaches to study wake fields are difficult and only a few measurements have been made. The non-reciprocal nature of the grain interaction of aligned grains has been demonstrated by Takahashi *et al.* [22].

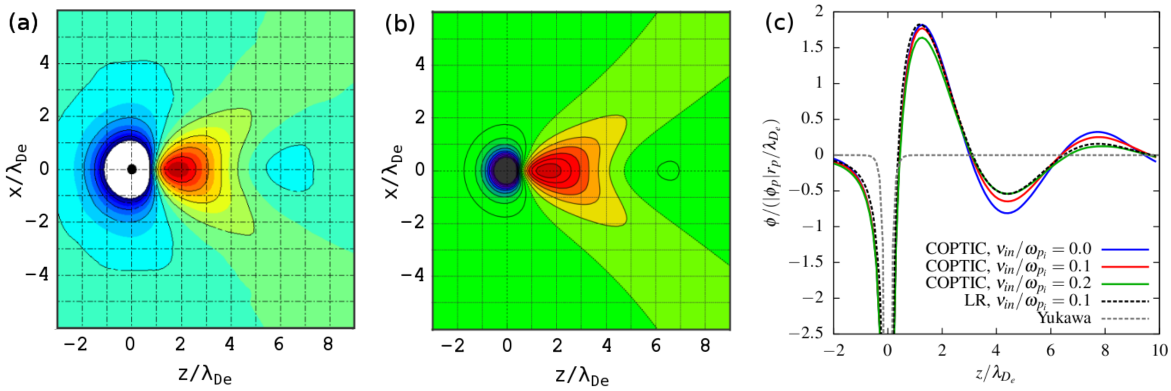

Fig. 1 Comparison of complementary numerical approaches: (a) full 3D PIC result from DiP3D for the wake field potential of a single dust grain (black dot, radius $a = 0.185\lambda_{De}$) at supersonic ion flow ($v_d = 1.5C_s$). The ion flow is from left to right and positive potential values are yellow or red while negative values are blue. The white area corresponds to very negative potential values. Despite the fact that the PIC simulations correspond to the nonlinear regime, the corresponding LR result in (b) reveals the same topological wake structure [31] (c) wake field potential along z-direction for $x = 0$ computed with LR-theory and with PIC simulations using COPTIC for different collision frequencies ($v_d = C_s$).

Melzer *et al.*. [23] showed that the attractive character of the wakes potential critically relies on ion collisions and gave a first estimation of the attractive force. The strong influence of the confinement on the grain arrangement was studied by Steinberg *et al.* [24] and Samarian *et al.* [41]. The ion focus strength has been studied by Hebner and Riley using binary collisions [25,26]. Unfortunately, most investigations so far assumed that the downstream grain neither affects the wake of the upstream grain nor is affected itself by the upstream grain. Basically ion drag forces and variations of grain charge were neglected. First experimental evidence that the grain charge changes for aligned grains was reported by Kong *et al.* [36]. Recently, Kroll *et al.* [28] studied the relaxation process in a system consisting of two grains and showed that the wake strongly alters the grain charge and Carstensen *et al.* developed a phase resolved resonance method [42] which allows to determine grain charges with high accuracy. Thus, the combination of advances in simulation and experiment now provides the possibility to compare results quantitatively. The following sections are devoted to this task.

3 Comparison of Theory and Simulation

In a first step, we compare the electrostatic potential around a single grain as obtained from LR-theory and from PIC simulations. Fig 1(a) shows the contours of the wake field potential of a dust grain which is placed in the origin of the plot (black dot). The collisionless plasma is streaming with supersonic ion velocity in positive z-direction, e.g. $v_d \geq C_s$ and $C_s = \sqrt{k_b T_e/m_i}$ (where m_i is the ion mass and T_e is the electron temperature). The plasma potential is computed using the DiP3D code. Clearly, a deep potential minimum is observed in direct vicinity of the grain. In x-direction, i.e. perpendicular to the flow, the potential is in good agreement with a Yukawa potential $\Phi_{z=0} \sim \exp\left(-x/\lambda_{De}\right)$ [31] which is supported by experiments [43]. In z-direction the potential strongly deviates from a statically screened Coulomb (Yukawa) potential. Downstream of the grain a pronounced potential maximum is found near $z = 2\lambda_{De}$. The potential maximum has a triangular shape and a Mach-cone like structure is formed. Thus, our PIC simulations nicely confirm previous findings e.g. [12]. Furthermore, Fig 1(b) shows that all topological features are well reproduced by the LR calculations. The potential maximum is found at $z = 2\lambda_{De}$ and the shape and the Mach cone angle agree as well. A more detailed comparison of DiP3d and LR results is given in a recent paper [31]. There it is shown that the peak positions generally agree well for supersonic ion flows. Even in the subsonic regime, where PIC codes are more noisy, a reasonable agreement is found. A quantitative comparison of the potential profile for a collisional plasma is given in Fig. 1(c). Again, the peak shape and wavelength agreement is very good. However, as LR uses the grain charge as an input parameter while in PIC simulations the charge is computed self-consistently, the amplitude of the LR results are rescaled according to the grain charge (see normalized ordinate in Fig. 1(c)). Nonetheless, the results

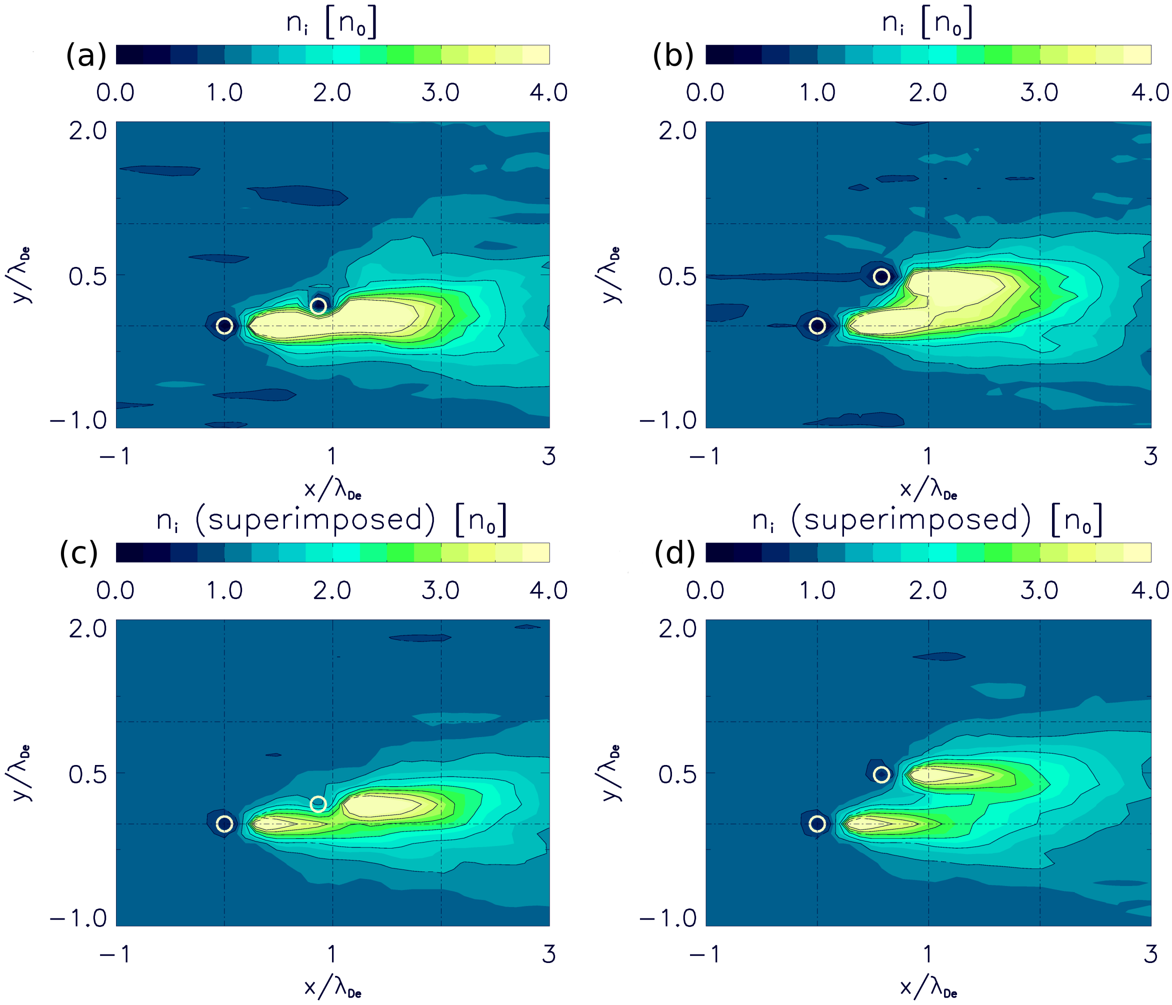

Fig. 2 Ion density (color coded) for two dust grains in a slightly supersonic ion flow ($v_d = 1.2C_s$). The grain positions are marked with white circles. (a) and (b) are self-consistent computations with the DiP3D code for (a) an almost aligned grain arrangement and (b) for a notable grain separation perpendicular to the ion flowing. In both cases the intergrain distance is λ_{De}. The panels (c) and (d) show the same situation but here the ion density is obtained from a superposition of wakes from isolated grains which were cacluted with DiP3d and account for a charge adjustment on the downstream grain.

clearly show that for a single grain LR and PIC simulations can yield comparable results even for relatively large grains (radius $a = 0.185\lambda_{De}$) [31].

For a complex plasma the calculations of the wake of a single grain is of basic interest, but equally important is the question how a second grain will modify the results. Fig. 2 shows the ion density for a system consisting of two dust grains (radius $a = 0.1\lambda_{De}$). The results are all obtained for a collisonless situation using the DiP3D code. While Fig. 2(a) and (b) are self-consistent computations, the panels (c) and (d) are superpositions of wakes computed for isolated grains with adjusted charges to match the conditions of panel (a) and (b). Thus, a comparison of Fig. 2(a) and (b) and Fig. 2(c) and (d) can be regarded as an estimate for the importance of nonlinear effects. Generally, the results agree well, but there are some differences. First, the ion focus region of the upstream grain is significantly stronger in (a) and (b). Second, the ion focus regions are not separated in panel (a) and (b). The relative deviations of panel (a) and (c) as well as for (b) and (d) can locally reach 50 percent. A comparison for plasma potential leads to similar results [31]. This implies that nonlinear effects are relevant in a situation with multiple grains, but for small grains these deviations are reasonable [21]. Thus, combined MD/LR simulations with an advanced grain-charging model are certainly a very promising approach to describe many experimental conditions.

For experiments [23, 25, 26, 41], this finding has consequences as well. The fact that the downstream grain modifies the ion focus of the upstream grain implies that the downstream grain cannot be used straight forward as a probe for the ion focus region of upstream grains. Only for a sufficient distance between the grains the assumption of linear superposition is generally justified.

4 Charging of grains in streaming plasmas

The grain charge is determined by the plasma conditions. The floating potential of the grain establishes in such a way that electron and ion currents balance. The influence of a single dust grain on the overall quasi-neutrality $n_e = n_i$ can be neglected and for collisionless plasma conditions OML-theory is widely used to compute the grain potential and finally the grain charge. In a streaming plasma the grain charge additionally depends on the

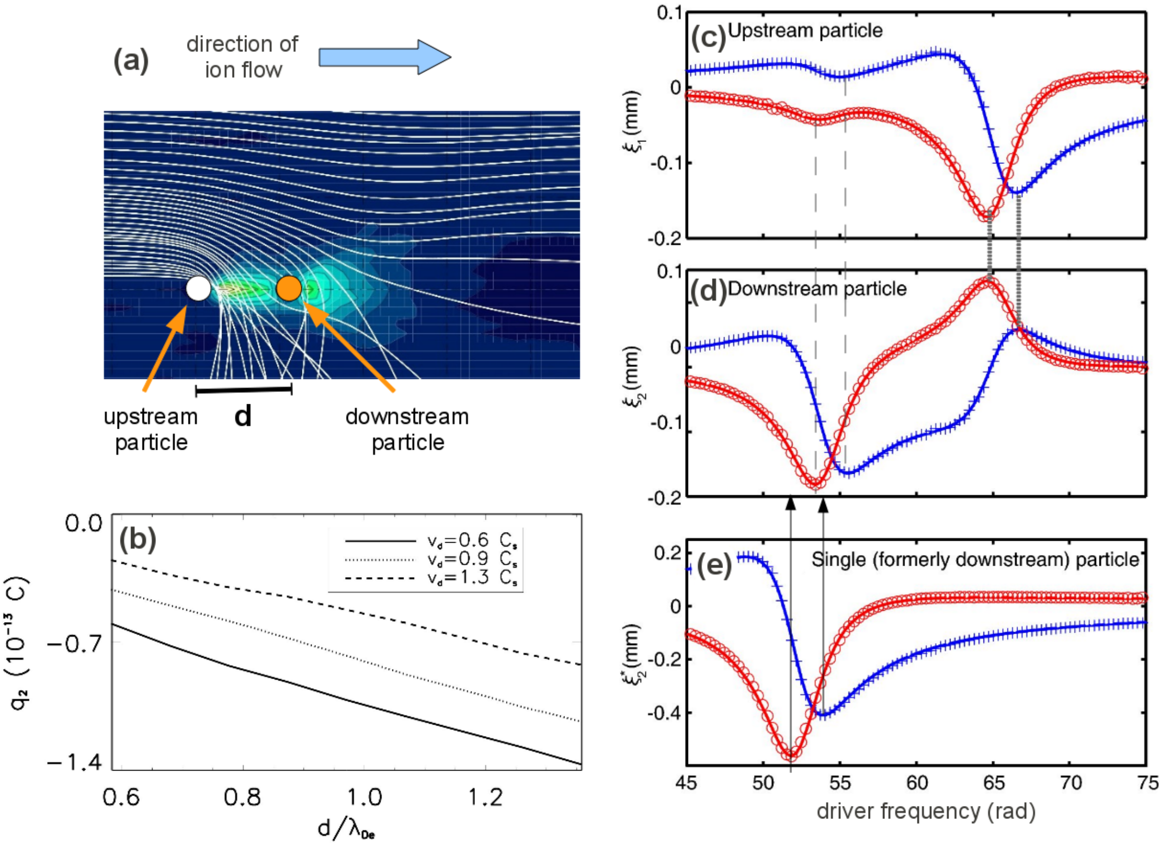

Fig. 3 Charge reduction of a grain in the wake of another grain: (a) ion density contour plot for two dust grains aligned in direction of the ion flow obtaind with DiP3D PIC-code. The distance between grains is d. The ion density is clearly enhanced downstream of each grain. The solid lines visualize collisionless ion trajectories. (b) shows the charge of the downstream grain q_2 as a function of d. The downstream grain charges negatively and its charge increases with increasing distance d. Compared to the charge $q_1 \approx -1.6 \times 10^{-13}$ C of the upstream grain a significant charge reduction is observed. (c,d) Resonance curves obtained in experiment with PRRM for two aligned grains, which are confined in the plasma sheath. Corresponding resonances of up- and downstream grain are indicated by dashed/dotted lines. (e) shows the resonance curve of a single (formerly downstream) grain. The resonance frequency has shifted to lower frequencies. Taking the coupling of the grains into account, this corresponds to a charge reduction for the downstream grain.

flow velocity v_d. As the cross-section for ion collection reduces with increasing v_d the electron and ion currents are only balanced at a more negative floating potential. The grain charges more negatively with increasing v_d. In collisonless PIC simulations the charging process is computed self-consistently and agrees well with OML-theory [33, 44, 45].

For two (identical) grains in a flowing plasma the situation is more sophisticated. Fig. 3(a) shows the ion density for a pair of grains, which are aligned in direction of the ion flow. In addition, trajectories of individual ions are plotted in white. Obviously, the upstream grain shadows the downstream grain but at the same time its negative charge deflects ions into the focus region. Thus, the current balance for the downstream grain differs completely from those of the upstream grain. Fig. 3(b) depicts the charge of the downstream grain as a function of grain distance d. Compared to the charge $q_1 \approx -1.6 \times 10^{-13}$ C of the upstream grain, the charge q_2 of the downstream grain is significantly reduced. A significant charge reduction is observed for both sub- and supersonic flow velocities. A systematic investigation of grain charges q_1 and q_2 as a function of inter-grain distance and flow velocity can be found in Ref. [45].

In experiments the Phase Resolved Resonance Method (PRRM) [42] has been developed to measure grain charges with high precision. The results shown in Fig. 3(c)-(e) are measured for two grains which are confined in the plasma sheath above a powered rf-electrode. A small sinusoidal bias modulation of the electrode drives vertical oscillations of the grains. Now, the grain positions at two distinct phase angles (red and blue) with respect to the driver signal are measured. This allows to determine phase and amplitude of the oscillation. The plots (c) and (d) show the grain positions of upstream and downstream grain as a function of the driver frequency ω. Based on a simple model for coupled harmonic oscillators including a external drive $F_{ext}(\omega)$ and neutral gas friction, the equation of motion for this system then reads

$$\ddot{\xi}_1 + 2\gamma_1\dot{\xi}_1 + \omega_1^2\xi_1 + D_{12}(\xi_1 - \xi_2) = \frac{F_{ext}(\omega)}{m_1} \tag{2}$$

$$\ddot{\xi}_2 + 2\gamma_2\dot{\xi}_2 + \omega_2^2\xi_2 + D_{21}(\xi_2 - \xi_1) = \frac{F_{ext}(\omega)}{m_2} \tag{3}$$

where the positions of the grains $\xi_j(t)$ are oscillatory solutions, i.e. $\xi_j(t) = A_j \exp i\omega t$. From Fig Fig. 3(c) and (d) it is seen, that the solutions (solid blue and red lines) fit the experimental data very well [29]. An essential feature of this oscillator model is that we allow for non-reciprocal forces between the grains. The reason for this is directly seen from plot (c) and (d). While the downstream grain shows two pronounced resonances at 53 rad/s and 64 rad/s the upstream grain shows only a pronounced resonance at 64 rad/s. The resonance at 53 rad/s is visible but weak. Therefore, the upstream grain clearly has a strong influence on the dynamics of the downstream grain, but not vice versa. This non-reciprocal character had been demonstrated earlier [22, 23] and is caused by the supersonic ion flow, which inhibits any upstream propagation of an ion density perturbation. In a second part of the experiment the upstream grain was removed and the downstream grain remained trapped. The discharge conditions were not changed and the resonance curves of the single grain was measured (see Fig. 3(e)). Obviously the resonances of the single downstream grain shifted to lower frequencies. Usually lower resonance frequencies are related to a lower charge to mass ratio, but here we have to compare the resonance of a single grain with the resonance of two coupled grains, where the coupling significantly influences the resonance frequencies. A detailed evaluation of the resonance frequencies found in Figs. 3(c-e) yields a reduction of charge for the aligned downstream grain of 22 percent. Repeating the same experiment but removing the downstream grain, the charge of the upstream grain does not change. Thus, the experiment proves a considerable charge reduction in the wake of other grains.

In addition, the analysis allows to determine the coupling constants for both oscillators. With $D_{12}/D_{21} = 5.5$ the ratio of the coupling constants is significantly larger than one, which means that the upstream grain strongly influences the downstream grain but not vice versa. This asymmetry has two reasons. The first is the supersonic ion flow. However, according to PIC simulations the ion flow alone only yields $D_{12}/D_{21} \approx 1.5$. Thus a second mechanism is required. In Fig. 3(b) the charge reduction was found to depend strongly on the intergrain distance d and that the charge gradient is to a good approximation linear. Taking into account that the equilibrium position of the grains is determined by a balance of gravity and electric field forces, the charge variation on the downstream grain caused by an oscillation of the upstream grain generates an imbalance of electric field forces and gravity and thus an effective enhancement of the coupling constant [29].

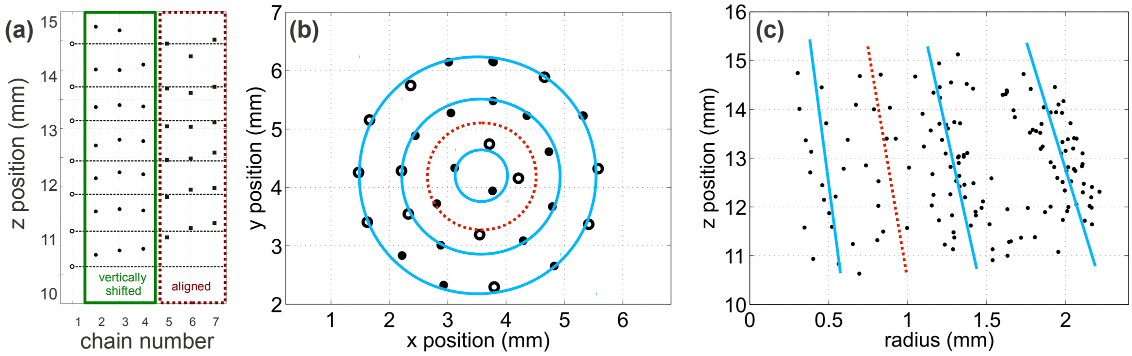

Fig. 4 Structural analysis of a 3D dust cloud consisting of 150 grains confined in a Yukawa ball trap. The grains arrange themselves in vertical chains. (a) The vertical positions of grains are shown for a reference chain (No. 1) and its six neighboring chains (No. 2-7). The neighboring chains are either horizontally aligned (No. 5-7) or vertically displaced by half an inter-grain distance (No. 2-.4). (b) shows the midplane of the cluster. Vertically displaced chains are indicated with open symbols.The chains clearly arrange in shells with an additional subshell close to the center (dashed line). (c) Radial distribution of grains with respect to their z-position. The shells shown in (b) are seen at all vertical positions.

Therefore, the experiments show that the force balance, coupling and charging of only two aligned grains is a sophisticated problem. This problem is further complicated if the grains are not aligned or if multiple grains are studied [46]. The experiments of Kroll *et al.* [28] in combination with PIC simulations [33] showed that the charging of grains dramatically changes as soon as they enter the wake region. Finally, drag forces have not been considered so far. Extensive PIC-simulations by Patacchini *et al.* [47] have shown that drag forces along the flow are positive for all relevant conditions. Thus, an upstream motion of grains as observed in MD simulations with LR-wake potentials (which neglect momentum transfer) [40] is not expected in experiments. Lapenta [9] and recently Piel [35] discussed the stability of chains of grains perpendicular to the ion flow. Both find that the drag forces have a stabilizing effect on chains of grains, but Piel's trajectory calculations show that the transverse momentum transfer originating from a loss of rotational symmetry for a displaced downstream grain gives the dominant contribution. The maximum of the transverse force is predicted for subsonic flows ($v_d \approx 0.6C_s$). A regime where the ion focus and thus its attractive force on a downstream grain is still weak. For sonic flows, drag forces are reported to be less important than the attractive wake field itself [48].

5 Dust clouds in streaming plasmas

Chains of grains are naturally the first step to understand the structural and dynamical properties of dust clouds. As today's simulations are just beginning to explore this regime [31, 40, 45, 46], the current understanding is founded on experimental results. Experiments under microgravity conditions [1] and in the laboratory [49–51] allow to confine dust clouds outside the sheath region. Especially for Yukawa balls [52] the structural and dynamical properties are well explored [3, 53]. Yukawa balls exhibit a pronounced concentric shell structure without any chain formation, and neither structural nor dynamical processes show indications for an attractive grain interaction. From discharge simulations the ion drift velocities are known to be well below the sound speed. Fig. 4 shows measurements of a dust cloud that is confined in a trap in which Yukawa balls form. Here, two observations differ with respect to Yukawa balls. The neutral gas pressure is just $p = 4\,\mathrm{Pa}$, which is a factor of ten below the reference value of typical Yukawa balls, and with a diameter of $20\,\mu\mathrm{m}$ the grains are five times larger. The resulting structure of this dust cloud has similarities with Yukawa balls. The shape is roughly spherical and the grains arrange in shells (Fig. 4(b,c)). However, in vertical direction a pronounced chain formation is observed (Fig. 4(a)), where neighboring chains are either horizontally aligned or vertically shifted by half an inter-grain distance (Fig. 4(a)). Additional experiments with two grains [28] and with normal mode analysis [54] at similar discharge conditions have demonstrated that the chain formation is clearly caused by an attractive potential below each grain. The grain dynamics are similar to those observed for two grains in the plasma sheath [24, 27] where the supersonic ion flow produces a strong ion focus region. However, for the experimental conditions described here supersonic ion flows are not expected. Thus, the experiments suggest that an attractive ion focus region exists even at subsonic conditions. This interpretation is supported by LR and PIC simulations [31,45] as well as the stability analysis of chains by Piel [35]. In all approaches large grains as well as a collisionless situation are beneficial for chain formation. The low neutral gas pressure results in a larger mean free path for ions and the larger grains result in a higher grain charge, which in turn deflects more ions into the wake region. However, this alone is not sufficient to finally prove attractive forces for subsonic flows, but recent experiments by Arp *et al.* [34] report chain formation in the plasma bulk region where the ion flow is definitely subsonic and Killer *et al.* [30] have shown that a weak ion focus affects the structure of elongated Yukawa balls. Thus, there is strong evidence that ion focusing and wake fields are relevant even in the subsonic regime.

6 Conclusion and Outlook

Ion flows change the interaction between dust grains significantly and introduce anisotropic order along the flow. Both LR and PIC simulations show the formation of a highly anisotropic wake region downstream of grains. The quantitative comparison revealed that the resulting wake calculated from both methods are in good agreement for individual grains. Also, the strength of the resulting ion focus downstream of the grain agrees well for PIC and LR calculations. To achieve a very good quantitative agreement an accurate charging model for the dust grains is essential to provide the correct input parameter for LR. For two grains, we found that a simple superposition of wake fields of individual grains is not fully reproduced by PIC simulations. However, for small grains ($a \ll \lambda_{De}$) and adjusted grain charges (see sect. 4) the linear superposition of wake potentials is certainly a reasonable approximation.

Experiments with two aligned grains and especially charge measurements with high precision have shown that the charge of the downstream grain is significantly reduced if the grain is placed in the wake region of the other grain. This important result is found to agree well with results from PIC simulations. The computed charge reductions as a function of the relative grain position explain the static as well as the dynamic properties of a confined system consisting of two grains in detail. In addition, the results clearly indicate, that the assumption of a fixed charge for all grains in a dust cloud is questionable as soon as the cloud is exposed to a substantial ion flow. Therefore, for simulations as well as the interpretation of experiments refined charging models are required which are capable to predict the individual charges in systems with many grains.

Furthermore our LR and PIC results show in very good agreement, that even for subsonic flows, wake field effects may play a role such as in the pre-sheath or even in the plasma bulk. Experiments on confined systems show the competition of spherical order which is induced by the external confinement on the one hand and chain formation under subsonic flow conditions on the other hand.

To conclude, further research is required to study the formation of wake fields and their influence on the static and dynamic properties of larger dust clouds. Especially the subsonic regime, many-particle effects and the role of grain confinement need to be investigated. A promising approach to tackle this multi-scale problem is the dynamical screening approach [32], which allows to study the structural and dynamical consequences of the wake field in large dusty plasma systems. Since wake fields also represent a source of free energy that can drive instabilities [7], this aspect will have to be included in future explorations of phase transitions in continuation of the investigations described in the companion paper Ref. [53]. Furthermore, the influence of magnetic fields on the structure of wake fields is of fundamental importance. Thus, extending the study of dust dynamics in the companion paper Ref. [55] into the large magnetic field regime opens a promising new direction of research.

Acknowledgements We gratefully acknowledge financial support by Deutsche Forschungsgemeinschaft under SFB-TR24 project numbers A2, A3, A5, and A7. Further, we thank I. H. Hutchinson for providing the data for a comparison of COPTIC and LR.

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